Ordonnancement multicoeurs optimisé de chaînes sur AFF3CT

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Starting point

• 1 filter = 1 task/bloc implemented (AFF3CT)

• How to split the computation for a given architecture while having good performances?

• For now: done manually by the user
Main issue

For a given multi-core architecture, with identical and homogeneous resources, automatically partition radio chain.

Objectives:

1. Maximize frame processing throughput
2. Minimize use of resources

What does it mean?

1. **Automatic parallelization of a task chain**
2. Automatic parallelization of **several task chain** on the same architecture
3. Automatic parallelization of a DAG
What about GNU Radio?

• For now:
  • Version 3.0
  • Thread Per Block (task) Scheduling
  • Thread management by the OS/runtime

• Future development:
  • Version 4.0
  • Allow more than one block per thread
  • Could use OTAC
State of the art: Pipeline workflow scheduling

- Task parallelism
- Pipeline parallelism
- Data parallelism
- Replicated parallelism

Parallelization operations available

Pipeline

Sequence 1  =  Stage 1
Sequence 2  =  Stage 2
Sequence 3  =  Stage 3

Pipeline user description
Parallelization operations available

Replication

Parallel task  Input socket  Thread
Sequential task  Output socket

Pipeline user description
Automatic parallelization of task chain

- $N$: number of tasks in the chain
- $P$: number of available resources
- $w_i$: weight of a task $\tau_i$

$\rightarrow$ Find optimal partitioning in terms of throughput for a given $P$

$w_2 = 5$

$N = 7$
Automatic parallelization of task chain

- $N$: number of tasks in the chain
- $P$: number of available resources
- $w_i$: weight of a task $\tau_i$
- $s_i$: $i^{th}$ stage
- $p_i$: number of resources dedicated to $s_i$

→ Find optimal partitioning in terms of throughput for a given $P$

\[ w_2 = 5 \]

$(s_0, p_0)$  $(s_1, p_1)$  $(s_2, p_2)$  $(s_3, p_3)$
Automatic parallelization of task chain

- $N$: number of tasks in the chain
- $P$: number of available resources
- $w_i$: weight of a task $\tau_i$
- $s_i$: $i^{e}$ stage
- $p_i$: number of resources dedicated to $s_i$
- Weight of the $i^{e}$ stage:
  
  $w(s_i, p_i) = \begin{cases} 
  \sum_{\tau \in s_i} w_i & \text{if } s_i \text{ stateful, } p_i \geq 1 \\
  \frac{1}{p_i} \sum_{\tau \in s_i} w_i & \text{if } s_i \text{ stateless, } p_i \geq 1 \\
  \infty & \text{otherwise}
  \end{cases}$

→ Find optimal partitioning in terms of throughput for a given $P$
Automatic parallelization of task chain

- $N$: number of tasks in the chain
- $P$: number of available resources
- $w_i$: weight of a task $\tau_i$
- $s_i$: $i^{th}$ stage
- $p_i$: number of resources dedicated to $s_i$
- Weight of the $i^{th}$ stage: $w(s_i, p_i)$

$$w(s_i, p_i) = \begin{cases} \sum_{\tau \in s_i} w_i & \text{if } s_i \text{ stateful, } p_i \geq 1 \\ \frac{1}{p_i} \sum_{\tau \in s_i} w_i & \text{if } s_i \text{ stateless, } p_i = 1 \\ \infty & \text{otherwise} \end{cases}$$

- $T^1 = \max_i w(s_i, p_i)$: the longest stage duration

→ Find optimal partitioning in terms of throughput for a given $P$

![Diagram showing task chain and stateful vs. stateless stages](image)
OTAC: **Optimal** maximal-packing for TAsk Chains

Goals:
1. Maximize the throughput i.e. minimize $T^{-1}$, the largest stage pipeline weight **automatically**
2. Minimize the number of used resources

Hypothesis:
- Identical and homogeneous resources
- No synchronization or data movement overhead


[https://hal.science/hal-04228117](https://hal.science/hal-04228117)
OTAC Algorithm

Two functions:

• **PROBE**
  • Find if, solution with target $T^{-1}$ and $P$ resources, is feasible

• **SOLVE**
  • Use PROBE and with a binary research on $T^{-1}$ find the best one

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**Algorithm 2:** SOLVE($N, P$)

**Result:** Minimal reciprocal throughput $T^{-1}$

1. $T_{\text{min}}^{-1} \leftarrow \frac{1}{P} \sum_{\tau \in T} w_{\tau}$;
2. $T_{\text{max}}^{-1} \leftarrow T_{\text{min}}^{-1} + \max_{\tau \in T} w_{\tau}$;
3. **while** $T_{\text{max}}^{-1} - T_{\text{min}}^{-1} \geq \frac{1}{P}$ **do**
   4. $T_{\text{mid}}^{-1} \leftarrow \frac{T_{\text{max}}^{-1} + T_{\text{min}}^{-1}}{2}$;
   5. **done**, $T_{\text{mid}}^{-1}$, $P$, $(n, p) \leftarrow$ PROBE($T_{\text{mid}}^{-1}$); /* TRUE if a valid solution is found */
   6. **if** done **then**
      7. $T_{\text{max}}^{-1} \leftarrow T_{\text{mid}}^{-1}$; /* $T^{-1}$ can only get smaller */
   8. **else**
      9. $T_{\text{min}}^{-1} \leftarrow T_{\text{mid}}^{-1}$; /* $T^{-1}$ can only get bigger */
   10. **end**
11. **end**
12. $T^{-1} \leftarrow T_{\text{max}}^{-1}$;
OTAC Algorithm

Find at most \( P \)-1 partitions, using **pipeline** and/or **replication** parallelism, minimizing the maximum sum of weight in all partition \( T^{-1} \).

\( N=9 \), \( P=7 \), \( T^{-1}=5 \)

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\( SF = \) stateful (task **not** replicable)  
\( SL = \) stateless (task replicable)
OTAC Algorithm

Find at most P-1 partitions, using **pipeline** and/or **replication** parallelism, minimizing the maximum sum of weight in all partition T⁻¹.

N=9, P=7, T⁻¹=5

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$W_{seq,1} = 12$

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\( N=9, \quad P=7 \)

\( T^{-1} = 5 \)

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**OTAC Algorithm**

Find at most $P-1$ partitions, using **pipeline** and/or **replication** parallelism, minimizing the maximum sum of weight in all partition $T^{-1}$.

$N=9$, $T^{-1}=5$, $P=7$

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$SF = \text{stateful (task not replicable)}$

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Find at most P-1 partitions, using **pipeline** and/or **replication** parallelism, minimizing the maximum sum of weight in all partition T^{-1}.

\[ \text{N}=9, \quad T^{-1}=5 \]
\[ \text{P}=7 \]

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Find at most P-1 partitions, using **pipeline** and/or **replication** parallelism, minimizing the maximum sum of weight in all partition \( T^{-1} \).

\( N = 9, \quad P = 7 \quad \Rightarrow \quad T^{-1} = 5 \)

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OTAC Algorithm

Find at most $P-1$ partitions, using pipeline and/or replication parallelism, minimizing the maximum sum of weight in all partition $T^{-1}$.

- \(N = 9\)
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P=7

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SF = stateful (task **not** replicable)
SL = stateless (task replicable)

N=9, P=7, $T^{-1}=5$
OTAC Algorithm

Binary search: find the best $T^{-1}$ for $P=7$

Lower Bound = $\lceil 28/7 \rceil = 4$

Upper Bound = $LB + 6 = 10$

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Total: 28

SF = stateful (task not replicable)
SL = stateless (task replicable)
OTAC Algorithm

Binary search: find the best $T^{-1}$ for $P=7$

Lower Bound = $\lceil 28/7 \rceil = 4$
Upper Bound = $\text{LB} + 6 = 10$

$T^{-1} = (\text{LB} + \text{UB})/2 = 7$

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</table>
**OTAC Algorithm**

Binary search: find the best $T^{-1}$ for $P=7$

- Lower Bound = $\lceil \frac{28}{7} \rceil = 4$
- Upper Bound = $LB + 6 = 10$

$T^{-1} = \frac{LB + UB}{2} = 7$

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$T^{-1} = 7$

$P$ Total: 5

$SF = \text{stateful (task not replicable)}$

$SL = \text{stateless (task replicable)}$
OTAC Algorithm

Binary search: find the best $T^{-1}$ for $P=7$

Lower Bound = 4
Upper Bound = 7

$T^{-1} = (LB+UB)/2 = 5.5$

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$T^{-1}=7$

$P$ Total: 6

SF = stateful (task not replicable)
SL = stateless (task replicable)
### OTAC Algorithm

**Binary search: find the best $T^{-1}$ for $P=7$**

- Lower Bound = 4
- Upper Bound = 7

$$T^{-1} = \frac{LB + UB}{2} = \frac{4 + 7}{2} = 5.5$$

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- **P Total:**
  - $T^{-1}=7$:
    - P Total: 6
    - 7
  - $T^{-1}=5.5$:
**OTAC Algorithm**

- **Binary search:** find the best $T^{-1}$ for $P=7$
- **Lower Bound = 4**
- **Upper Bound = 5.5**

$$T^{-1} = \frac{{\text{LB} + \text{UB}}}{2} = 4.75$$

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- **$T^{-1}=7$**
- **$P$ Total: 6**

- **$T^{-1}=5.5$**
- **$P$ Total: 7**

**SF = stateful (task not replicable)**
**SL = stateless (task replicable)**
**OTAC Algorithm**

Binary search: find the best $T^{-1}$ for $P=7$

Lower Bound = 4

Upper Bound = 5.5

$T^{-1} = (LB + UB) / 2 = 4.75$

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$T^{-1} = 7$

$P$ Total: 6  ✔

$T^{-1} = 5.5$

$P$ Total: 7  ✔

SF = stateful (task not replicable)
SL = stateless (task replicable)
**OTAC Algorithm**

Binary search: find the best $T^{-1}$ for $P=7$

- Lower Bound = 4
- Upper Bound = 5.5

$T^{-1} = (LB+UB)/2 = 4.75$

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**P Total:**

- $T^{-1}=7$:
  - $P$: $6$ ✓
- $T^{-1}=5.5$:
  - $P$: $7$ ✓
- $T^{-1}=4.75$:
  - $P$: $8$ x
**OTAC Algorithm**

Binary search: find the best $T^{-1}$ for $P=7$

Lower Bound = **4.75**
Upper Bound = **5.5**

$T^{-1} = \frac{\text{LB} + \text{UB}}{2} = \frac{4.75 + 5.5}{2} = 5.125$

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$T^{-1}=7$

| P     | 3 | 1 | 2 |

$T^{-1}=5.5$

| P     | 3 | 1 | 1 | 2 |

$T^{-1}=4.75$

| P     | 4 | 1 | 1 | 2 |

**Note:**

- **SF** = stateful (task **not** replicable)
- **SL** = stateless (task replicable)
# OTAC Algorithm

- **Binary search**: find the best $T^{-1}$ for $P=7$
- **Lower Bound**: 4.75
- **Upper Bound**: 5.5
- **$T^{-1} = \frac{LB+UB}{2} = 5.125$**

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</table>

- $T^{-1}=7$
- $P$ | 3 | 1 | 2

- $T^{-1}=5.5$
- $P$ | 3 | 1 | 1 | 2

- $T^{-1}=4.75$
- $P$ | 4 | 1 | 1 | 2

- $T^{-1}=5$
- $P$ | 2 | 1 | 1 | 1 | 2

**P Total**: 6 ✅, 7 ✅, 8 ✗, 7 ✅
**OTAC Algorithm**

Binary search: find the best $T^{-1}$ for $P=7$

- **Lower Bound** = 4.75
- **Upper Bound** = 5.125

UB-LB = 0.375 < 1 with non-integer values < $1/P$

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- **$T^{-1}=7$**
  - $P$ = [3, 1, 2]

- **$T^{-1}=5.5$**
  - $P$ = [3, 1, 1, 1, 2]

- **$T^{-1}=4.75$**
  - $P$ = [4, 1, 1, 1, 2]
  - **Total**: 8
  - **Result**: ❌

- **$T^{-1}=5$**
  - $P$ = [2, 1, 1, 1, 1, 2]
  - **Total**: 7
  - **Result**: ✔️
OTAC Algorithm

Binary search: find the best $T^{-1}$ for $P=7$

Lower Bound = 4.75

Upper Bound = 5.125

$UB-LB = 0.375 < 1$ with non-integer values $< 1/P$

<table>
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<th>Index</th>
<th>State</th>
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For this specific problem with $P=7$, $T^{-1}=5$
Evaluation

1. Synthetic chains
2. Concrete use-case: DVB-S2
Evaluation: synthetic chains

Workload:
• 1000 randomly generated chains
• 20 tasks each
• Computational time [100,35000] µs
• Stateless ratio $SR \in [0, 1]$

Variation of SR from 0 to 1 with step 0.1
$P = \{5, 10, 15, 25\}$
Evaluation: synthetic chains

Comparison to other algorithms?

• **Nicol**: optimal algorithm for pipelining only
• **RB**: recursived-bipartitioning-based scheduler
• **RBm**: as RB + merge neighboring stateless stages together
• **RPT**: one resources per task, if N>P, fuses stages together
• **RPTm**: as RPT + merge neighboring stateless stages together
Evaluation: synthetic chains

Comparison to other algorithms?

- **Nicol**: optimal algorithm for pipelining only
- **RB**: recursived-bipartitioning-based scheduler
- **RBm**: as RB + merge neighboring stateless stages together
- **RPT**: one resources per task, if N>P, fuses stages together
- **RPTm**: as RPT + merge neighboring stateless stages together
Number of solutions of different qualities for different schedulers, numbers of processors, and stateless ratios compared to OTAC.

- **Optimal**: maximum throughput with minimum number of resources.
- **Excess**: maximum throughput with extra resources.
- **Suboptimal**: suboptimal throughput.
Simulation: histograms of number of resources used (P)

Schedules computed by:
- OTAC
- RBm
- Nicol

Configurations:
- Left: P = 20 and SR = 0.5
- Right: P = 20 and SR = 0.9
Evaluation: concrete use case DVB-S2

**Hardware & Software environment**
- 2 x 18 Intel Xeon Skylake, 3.2 GHz
- No hyperthreading
- AFF3CT v3.0.2
- gcc v12.2.0 - Python v3.8.0
- Threads pinned onto core with hwloc v2.7

**DVB-S2 configuration**
- Satellite transmission of video contents
- Reception chain at transmission time
- Frame packet of 16 frames
- 1.28 GB decoded by execution
- Noise 4 dB
Evaluation: concrete use case DVB-S2

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Simulated T</th>
<th>T average</th>
<th>P</th>
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<tr>
<td>OTAC</td>
<td>47.17</td>
<td>43.86</td>
<td>18</td>
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<tr>
<td>RPTm</td>
<td>26.21</td>
<td>25.26</td>
<td>18</td>
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<tr>
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<td>Baseline</td>
<td>47.17</td>
<td>42.3</td>
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Simulated and average throughput, number of resources needed for the best results achieved.
Conclusion

Summary:
• Optimal algorithm maximizing throughput and minimizing resources

On-going:
• Automatic parallelization of several task chain on the same architecture

To go further:
• Manage more complex graphs (loop, condition, DAG)
• Is it transferable for neural network training ? => internship
• On-the-fly noise variation